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BRIEF REPORTS

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Persistent self-organization of sandpiles

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We have studied the avalanche fluctuations resulting from the grain-by-grain quiescent perturbation of real sandpiles of various sizes. Contrary to previous reports, we find the attribute of self-organization, namely, a power-law distribution of avalanche sizes, to be the generic behavior, independent of system size. However, as the system size is increased, the power-law behavior is supplemented by uniformly large, periodic avalanches that carry away the dominant fraction of the total avalanched mass. Additionally, we find that a scale invariance in pile-mass fluctuations depends on the ratio of pile to grain diameters.

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Avalanches in an evolving sandpile have become a provocative paradigm for the phenomenon known as self-organized criticality (SOC) [1]. At least two distinct types of sandpile experiments have emerged to test for SOC. In one experiment [2], a 5-cm radius, semicircular drum of sand (grain diameter 0.54 mm) is slowly and continuously inclined. The flow of sand over the drum's edge is measured between capacitor plates. The main results, contrary to the tenets of SOC, were sharply peaked distributions of avalanche interval and duration. The sharply peaked avalanche duration of approximately 1.5 s resulted from large, system-spanning events. These were interpreted as a hysteresis effect whereby the pile's inclination angle would exceed the metastable angle of repose by about 2° before discharging a major avalanche that would return the pile to its metastable state. Such a sharp peak in avalanche duration implies avalanche sizes that are certainly not power-law distributed over a broad range. The experimenters also claimed that a systematic reduction in the hysteresis (effected by vibrating the pile) did not recover any universal power law. In another experiment [3], individual grains were intermittently dropped on the apex of a conical sandpile and the resulting pile-mass fluctuations measured. A personal-computer (PC) interfaced microbalance continuously monitored the sandpile mass; any grains falling off the pile were diverted from the weighing platform and indicated avalanche events. For pile bases ranging from 10 to 50 grain diameters, avalanche size distributions were found to be power laws and subject to finite-size scaling. In contrast, piles on the order of 100 grain diameters were claimed to have supported *only* the uniformly large, periodic avalanches similar to those reported in Ref. [2], and thus not subject to SOC.

Compelled to verify that "small" and "large" piles truly behave differently, we have carried out an experiment based on the apparatus and techniques described in Ref. [3]. The silicon dioxide sand used in this experiment had a mean grain mass of 0.7 mg for the 0.8-mm diameter and 0.1 mg for the 0.4-mm diameter. We employed a slowly rotating glass funnel to dispense an average flow of one grain every 4 s, while sampling the balance every 0.5 s with a PC. Any time the scale measured an absolute change equal to or greater than the mass of a single grain, a "dropping event" and current mass values were recorded. A one grain on, one grain off event is not recorded. In order to drop the grains as quiescently as possible, we strived to maintain the dropper's lateral precession at 1 mm and height above the sandpile's apex at 5 mm.

In Fig. 1 we plot the pile-mass histories for piles with base diameter L_p ranging from 2 to 8 cm. These histories, spanning 20 000 dropping events each, comprise the data from which all subsequent information is derived. Any decrease of pile mass represents an avalanche. The mean grain diameter L_g was 0.8 mm, implying system sizes ranging from 25 to 100 grain diameters. Figure 1 depicts an evolution from the apparently random fluctuations for the smallest pile to the smooth, periodic behavior in the largest. The 6-cm pile appears to represent a crossover from one behavior to the other. Because the smallest-sized avalanches would become decreasingly visible on the mass scale of ever larger piles, we examined the fine structure of mass histories for the 4- and 8-cm piles. Figures 2(a) and 2(b) show 100-event segments of

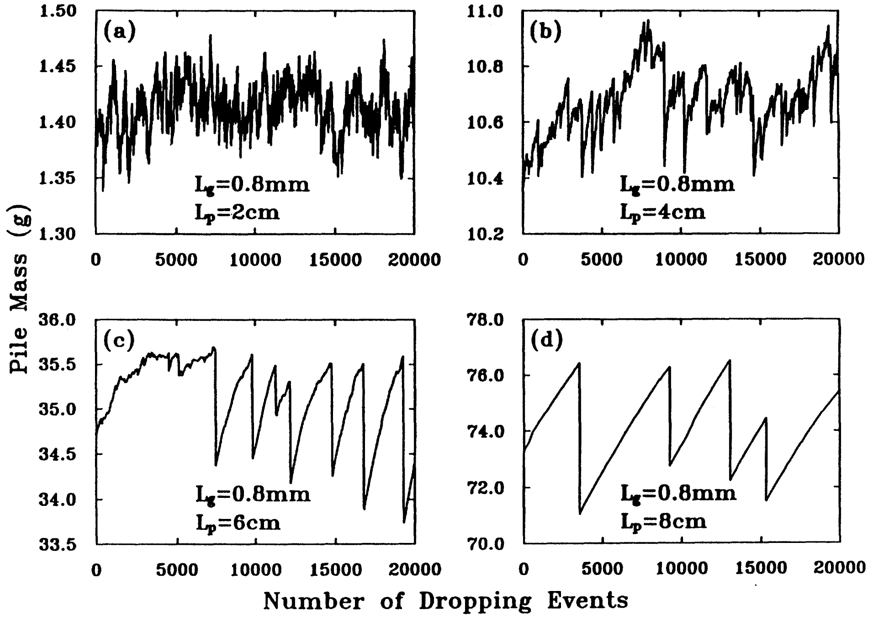

FIG. 1. Pile-mass histories as a function of dropping events for grains of mean diameter 0.8 mm. The pile base diameter L_p ranges from 2 to 8 cm.

the mass histories in Figs. 1(b) and 1(d), respectively. These figures exhibit a comparable abundance of small avalanches despite the apparent smoothness of the entire 8-cm pile-mass history in Fig. 1(d) compared to the 4-cm pile in Fig. 1(b).

The frequency of occurrence of avalanches of size S grains, $P(S)$, shown in Fig. 3, confirms this qualitative observation. Here the data points have been binned in order to obtain a relative error in probability no greater than 40%, and then averaged to indicate the most likely avalanche size in a given range. The binning consists in assigning the same probabilty to all events with an avalanche-size range in which at least five events occur. This probability is given by the frequency of occurence of

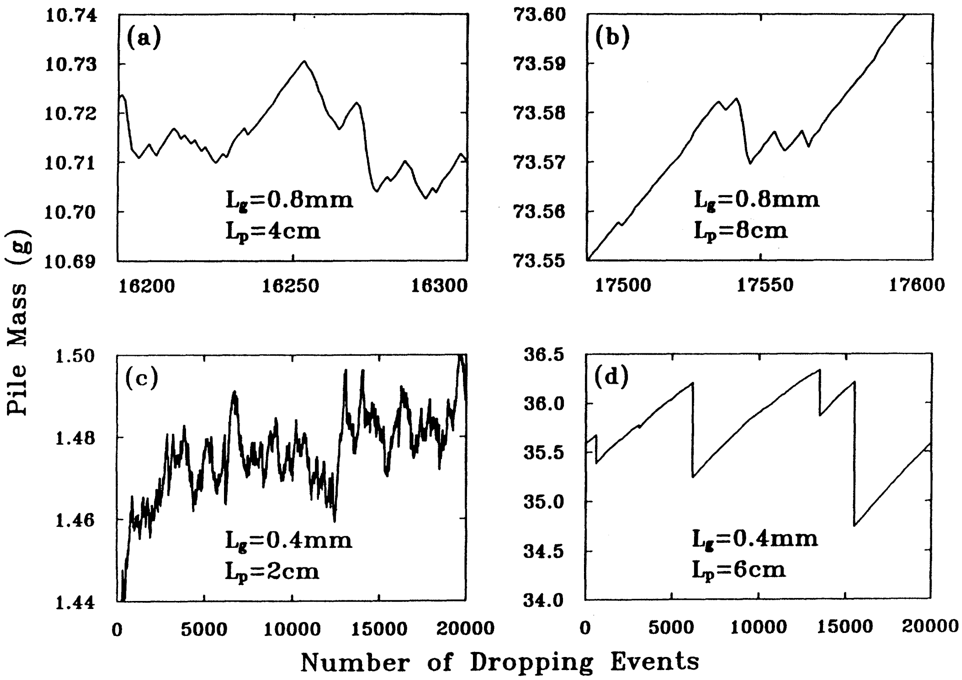

FIG. 2. Pile-mass histories as a function of dropping events. In (a) and (b) we show magnifications of Figs. 1(b) and 1(d), respectively. In (c) and (d) we show comparable fluctuations to Figs. 1(b) and 1(d) with $L_g = 0.4$ mm.

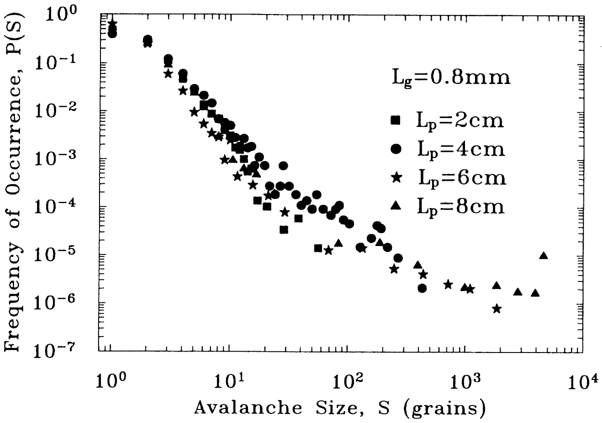

FIG. 3. Normalized frequency of occurrence $P(S)$ as a function of the avalanche size S, in grains. The mean grain diameter is 0.8 mm with pile diameters ranging from 2 to 8 cm. The points have been binned and averaged as indicated in the text.

five events weighted by the width of that avalanche-size range. To each avalance-size range a single probability was assigned at the centroid of that range. This plot indicates a power law, $P(S)\sim S^{-2.2}$ for avalanches between 2 and 20 grains, independent of pile sizes. In contrast to what might be inferred from Fig. 1(d), the smallest avalanches, with power-law size distribution, persist as the most frequently occurring in the largest pile. However, the larger piles differ from the smaller by showing high mass tails that are at least 1000 times less likely than a 2- or 3-grain avalanche.

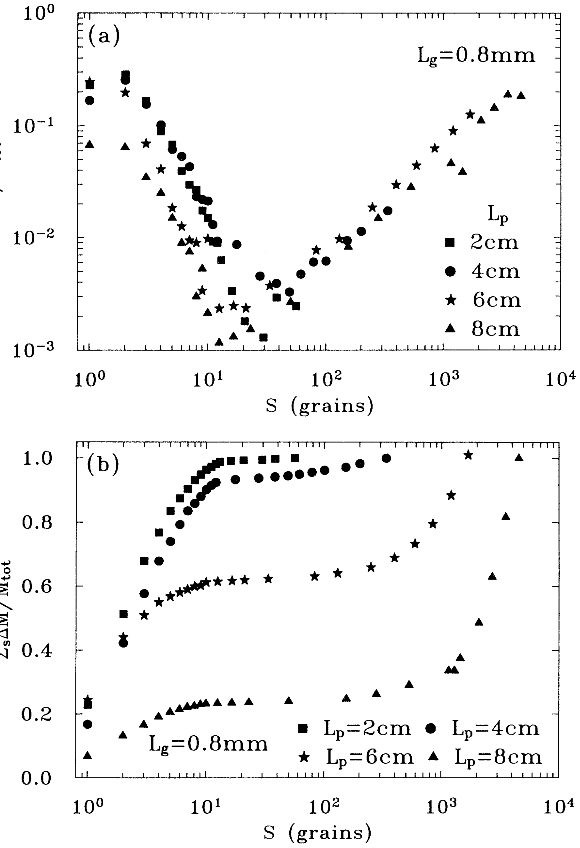

FIG. 4. (a) Normalized fraction of total avalanched mass removed by avalanches of size S. (b) Normalized fraction of total avalanched mass removed by avalanches less than or equal to S.

Systems exhibiting SOC should also show finite-size scaling of the form [1]

$$P(S,L_p)=L_p^{-\beta}g(SL_p^{-\nu}) ,$$

where $P(S,L_p)$ is the probability of an avalanche of size S for a pile with base diameter L_p and g is a universal scaling function. The constraint $\beta=2\nu$ ensures that the average pile mass is stationary [4]. However, no such scaling law was found to simultaneously scale both small and large avalanches of this experiment.

Although the smaller avalanches (1 to 20 grains) persist as the most frequently occurring, the fraction of total mass removed by them diminishes with increasing pile size. This fact is evident from Fig. 4(a), where the fraction of total mass removed $\Delta M/M_{\text{tot}}$ is plotted against avalanche size S. The fraction of total avalanched mass $\Delta M/M_{\text{tot}}$ has been obtained from multiplying $P(S)$ by the ratio of the mass in an avalanche of size S to the mean avalanche mass. In Fig. 4(b), the fraction of total mass removed by avalanches of size less than or equal to S reveals that as L_p increases, the largest avalanches carry away a sharply increasing fraction of the total avalanched mass. For the 6- and 8-cm piles it is also evident that the avalanches in the mid-sized range do not contribute significantly to the cumulative avalanched mass.

Because the durations of even the largest avalanches (≈ 1 s) were small compared to the average interval between dropping events (≈ 4 s) and individual avalanche durations were not measured, the mass power spectra of Fig. 5 reveal temporal information only in the low- to middle-frequency regimes. For the largest pile, a well-defined power maximum occurs at a frequency (2×10^{-4} events^{-1}) corresponding to about 5000 dropping events: the average buildup interval for a large avalanche. These uniformly large, periodic avalanches are anticorrelated,

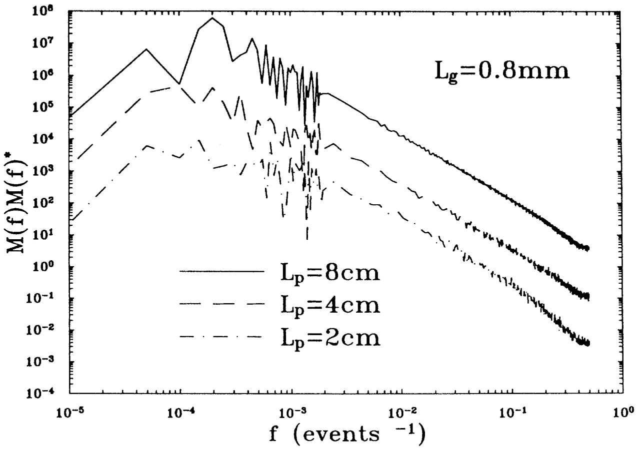

FIG. 5. Power spectra of piles of various sizes. Data at frequencies above 10^{-3} events^{-1} have been smoothed.

i.e., a major avalanche will most certainly be followed by a large time interval of pile-mass increase. The mid-frequency regime (10^{-3}–10^{-1} events^{-1}) falls off as f^{-2} and indicates that any correlation between avalanches of various sizes is short lived. However, Fig. 1(c) indicates that the major avalanche events are typically preceded by smaller ones and the exact correlation for such phenomena is worthy of future study.

Figures 2(c) and 1(b) show that mass fluctuations for a 2-cm pile of 0.4-mm grains are comparable to and scale invariant with those for a 4-cm pile of 0.8-mm grains. In this case $L_p/L_g=50$ is a constant for each mass history. Figures 2(d) and 1(d) show that when comparing the 6-cm pile of 0.4-mm grains to the 8-cm pile of 0.8-mm grains, which exhibit similar mass fluctuations, L_p/L_g is no longer constant. Why the constancy of L_p/L_g is not maintained for comparable fluctuations between the 6- and 8-cm piles is not well understood, but may be dependent on the threshold instabilities that generate the major, periodic avalanches.

In conclusion, we find that *all* system sizes studied, which include the relevant range in Ref. [3], support a predominant number of smaller, power-law-distributed avalanches. As the system size is increased, the power-law distribution is supplemented by uniformly large and periodic avalanches that remove a sharply increasing fraction of total avalanched mass. Since the fraction of total avalanched mass carried away by the smaller avalanches is negligible in larger piles, we conclude that, while SOC may be a finite-size effect, these smaller avalanches persist as the most frequently occurring even in the larger piles. The onset of domination by major avalanches depends on both the size of the pile and its grains.

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